Berry phases in superconducting transitions

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I generalize the concept of Berry's geometrical phase for quasicyclic Hamiltonians to the case in which the ground state evolves adiabatically to an excited state after one cycle, but returns to the ground state after an integer number of cycles. This allows to extend the charge Berry phase γ_c related to the macroscopic polarization, to many-body systems with fractional number of particles per site. Under certain conditions, γ_c and the spin Berry phase γ_s jump in π at the boundary of superconducting phases. In the extended Hubbard chain with on-site attraction U and nearest-neighbor interaction V at quarter filling, the transitions detected agree very well with exact results in two limits solved by the Bethe ansatz, and with previous numerical studies. In chains with spin SU(2) symmetry, γ_s jumps when a spin gap opens.
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In the last decade, Berry phases have caused a great deal of interest in a variety of fields in physics. Applications of this concept to condensed matter began with the study of Zak of the dynamics of a Bloch electron as its wave vector $\mathbf{k}$ changes adiabatically due to an external perturbation until it reaches $\mathbf{k+G}$ where $\mathbf{G}$ is a reciprocal lattice vector [1]. Although in this case the Hamiltonian which describes the evolution of the periodic part of the Bloch function is noncyclic, the initial and final points are related by a gauge transformation and a Berry phase can be defined for each band [2], which describes the center of gravity of the density of the Wannier function, or Wyckoff position [1,3]. Later progress, showed that changes in the macroscopic polarization of a band insulator in an independent particle approximation, are proportional to changes in a Berry phase, which I call γ_c [4,5]. The formalism was generalized to the many-body case [6,7] extending previous derivations for quantized charge transport [8], and was applied to study a ferroelectric transition in a strongly correlated model [9].

Recently Gagliano and me have introduced the spin Berry phase γ_s, and have calculated γ_c and γ_s for the first time in a gapless metallic phase [10,11]. The model was an extended Hubbard chain with correlated hopping, and due to the presence of inversion symmetry in it, γ_c and γ_s are quantized and can only have the values 0 and π. The topological quantum numbers γ_c/π and γ_s/π were used as order parameters to construct a phase diagram, separating the three phases of the model: charge-density wave (CDW), spin-density wave (SDW) and the metallic phase. The presence of anomalous flux quantization [10], and additional arguments [11], suggest that this phase is superconducting (S), possibly of triplet character. This work opened the possibility to study phase diagrams in strongly-correlated systems, by looking at topological transitions, which are sharp even in systems of finite size. However, all calculations of γ_c and γ_s in many-body systems, were so far restricted to an integer number of particles per site. This restriction is too severe, particularly if one is interested in superconductors.

In this Letter, I extend the definition of the Berry phase to the case in which it is necessary to perform more than one cycle in a parameter space, while the Hamiltonian ends at a point related to the starting one by a gauge transformation, in order for the system to return to a state equivalent to the initial ground state. At first sight, this seems to contradict the adiabatic theorem, since there should be a crossing of energy levels before the first cycle ends. However, this crossing is not essential if the matrix element of the Hamiltonian between the states involved vanishes, as it is the case if both states differ in some quantum number. While it seems difficult to imagine a quantum-mechanical system with these properties, the generalization below of the charge Berry phase γ_c to systems with fractional number of particles per site is an example. Calculating γ_c numerically in a system of 12 sites, I obtain the phase diagram of the extended Hubbard chain for 1/2 particles per site, negative U and any V. The results are compared with previous numerical ones, and with analytical ones obtained mapping the model in the limits $|U| \gg t, |V|$ and $|U|, V \gg t$ to Bethe ansatz solvable cases through appropriate canonical transformations. In one dimension (1D), for SU(2) symmetric systems, I show that the spin Berry phase jumps at the boundary between dominating singlet and triplet correlations at large distances, for any filling.

For simplicity I restrict the discussion to 1D and one band. Extension to the general case is straightforward. I consider a system of L sites and number of particles N with $N/L = n/l$, and n/l irreducible. The creation operators satisfy arbitrary boundary conditions for both spins $\bar{c}^\dagger_{j+L\sigma} = e^{i\Phi_\sigma}\bar{c}^\dagger_{j\sigma}$, and the Hamiltonian $\bar{H}(\Phi_\uparrow, \Phi_\downarrow)$

conserves number of particles, z component of the total spin, and is invariant under translations. This invariance allows to define weighted irreducible representations of the translation group characterized by the total wave vector $\bar{K} = K + (N_\uparrow \Phi_\uparrow + N_\downarrow \Phi_\downarrow)/L$, where N_σ is the number of particles with spin σ and $K = \text{integer} \times 2\pi/L$ is one of the allowed wave vectors for periodic boundary conditions (PBC) [12]. Using the gauge transformation $c^\dagger_{j\sigma} = e^{-ij\Phi_\sigma/L}\bar{c}^\dagger_{j\sigma}$, the Hamiltonian is converted into one in which the fluxes Φ_σ are distributed equally in each link $H(\Phi_\uparrow, \Phi_\downarrow)$ (the hoppings acquire a phase [10]), with creation operators satisfying PBC $c^\dagger_{j+L\sigma} = c^\dagger_{j\sigma}$. Note that while $\bar{H}(0,0) = \bar{H}(\pm 2\pi, \pm 2\pi)$, $H(0,0) = \bar{H}(0,0) \neq H(\pm 2\pi, \pm 2\pi)$ [2]. To generalize γ_c (γ_s), the idea is to start from the ground state $|g_K(\Phi_\uparrow, \Phi_\downarrow)\rangle$ of $H(\Phi_\uparrow, \Phi_\downarrow)$ for wave vector K and fluxes Φ_σ which minimize the energy, and follow the phase of $|g_K\rangle$ as the Φ_σ are shifted adiabatically by the same amount, with the same (opposite) sign, until $|g_K\rangle$ reaches a state equivalent to the initial one (the same eigenstate of $\bar{H}$ except for a phase). I discuss first γ_c. It can be defined as $\gamma_c = i\int_0^{2\pi l} d\Phi \langle g_K(\Phi,\Phi)|\frac{\partial}{\partial\Phi} g_K(\Phi,\Phi)\rangle$, or in the numerically gauge invariant form, discretizing the interval $0 \leq \Phi \leq 2\pi l$ into M points $\Phi_i = 2\pi l i/M$:

$$\gamma_c = -\lim_{M\to\infty} \text{Im} \ln \Pi_{i=0}^{M-2} \langle g_K(\Phi_i,\Phi_i)|g_K(\Phi_{i+1},\Phi_{i+1})\rangle \times \langle g_K(\Phi_{M-1},\Phi_{M-1})|e^{i\frac{2\pi l}{L}\sum_{j\sigma} j c^\dagger_{j\sigma} c_{j\sigma}}|g_K(0,0)\rangle \quad (1)$$

where the last ket is $|g_K(2\pi l, 2\pi l)\rangle$ calculated using the gauge transformation from the eigenstate $|g_K(0,0)\rangle$ of $\bar{H}(0,0)$. In practice, γ_c can be calculated to four digits accuracy using $M \sim 6$ adequately chosen points.

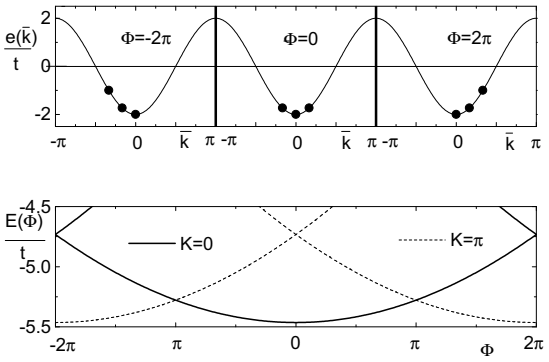

FIG. 1. Top: Scheme of the evolution of the ground state with flux Φ for a non interacting tight-binding model, showing the doubly occupied one particle states $\bar{k}$ (solid circles) in the one-particle energy dispersion $e(\bar{k})$. Bottom: Energy as a function of flux of the low-lying energy levels.

In previous cases, $l = 1$ and the integration was restricted to one cycle of $\bar{H}$ ($0 \leq \Phi \leq 2\pi$). However, while K is kept fixed, $\bar{K}$ evolves from K to $K + 2\pi n/l$ in this cycle. Since when $l \neq 1$ these $\bar{K}$ are not equivalent, the initial and final states are orthogonal and a Berry phase, even in its more general form [13], cannot be defined for this circuit. In Fig. 1 I illustrate the evolution of the ground state with Φ for $L = 12$, $n/l = 1/2$ for the case of a non-interacting nearest-neighbor (NN) tight-binding Hamiltonian (Eq. (3) for $U = V = 0$). The minimum energy corresponds to $K = \Phi = 0$. The adiabatic continuation of this state in the representation of H as Φ changes is always the same $|g_0(\Phi,\Phi)\rangle = \prod_\sigma c^\dagger_{-\pi/6\sigma} c^\dagger_{0\sigma} c^\dagger_{\pi/6\sigma}|0\rangle$, where $c^\dagger_{k\sigma}$ is the Fourier transform of $c^\dagger_{j\sigma}$. In the representation of $\bar{H}$, each one-particle $\bar{k}$ is shifted by Φ/L with respect to the corresponding k. It is easy to see that the energy of $|g_0(\Phi,\Phi)\rangle$ is $E_0(\Phi) = E(0)\cos(\Phi/L)$, where $E(0)$ is the energy for $\Phi = 0$ [12]. As Φ increases from 0 to 2π, each $\bar{k}$ goes to the next allowed wave vector for PBC, and $|g_0\rangle$ evolves to an excited state with $\bar{K} = \pi$. For Φ near 2π, the ground state is $|g_\pi(\Phi,\Phi)\rangle = \prod_\sigma c^\dagger_{-\pi/3\sigma} c^\dagger_{-\pi/6\sigma} c^\dagger_{0\sigma}|0\rangle$ with $K = \pi$. The crossing occurs for $\Phi = \pi$ and the ground state energy is of course periodic with period 2π. This crossing does not affect the adiabatic theorem, since the states $|g_0\rangle$ and $|g_\pi\rangle$ have different K. In contrast, $|g_0\rangle$ for $\Phi = \pm(2\pi - 0^+)$ represent orthogonal eigenstates of $\bar{H}$ with the same $\bar{K} = 0$ (see Fig. 1) and γ_c is undefined (one of the factors in Eq. (1) vanishes). However, since both states have the same quantum numbers, they should hybridize when interactions are present, removing the degeneracy at $\Phi = \pm 2\pi$. In general, for any L and $n/l = 1/2$, the above mentioned states differ in four particles displaced from the neighborhood of the Fermi point $-\pi/4$ to near the other Fermi point $\pi/4$. Umklapp processes of this type are generated in second and higher order perturbation theory in the interaction, and were studied previously [14]. For other fractional occupancies the situation is similar. The nature of the resulting thermodynamic phase as well as the resulting value of γ_c depends on the detailed form of the interaction.

I have reexamined the relation between γ_c and the macroscopic polarization in the present case, following the derivation done previously for $l = 1$ in the many-body case [6–8]. Essential for the extension is to work in a subspace with definite K, in such a way that for all Φ the ground state is separated from the excited states with the same quantum numbers. I obtain that changes in polarization ΔP are related to the corresponding changes in γ_c when some parameter of the Hamiltonian varies by:

$$\Delta P = (e/2\pi l)\Delta\gamma_c \text{ (modulo } e/l). \quad (2)$$

The extra factor l in the denominator is simply due to the l times larger interval of integration in the definition of γ_c. If $H(0,0)$ has inversion symmetry, $H(\Phi_\uparrow, \Phi_\downarrow)$ is transformed to $H(-\Phi_\uparrow, -\Phi_\downarrow)$ under inversion. As a consequence $\gamma_c = -\gamma_c$ (modulo 2π). This means that in systems with inversion symmetry $\gamma_c = 0$ or π (modulo

2π) as in previous cases [1,3,7,10].

The values of γ_c and γ_s are easy to predict for systems and parameters in which the particles are localized or the relevant kinetic terms are not affected by the flux. Examples are the CDW and SDW phases with maximum order parameter at half filling, for which $\gamma_c = \gamma_s = 0$ and $\gamma_c = \gamma_s = \pi$ respectively [10]. Another examples which are of interest because of its competition with S states, are states with phase segregation (PS) in which the particles group together. Let us take a chain of $L = 4N_\sigma$ sites in which the first N_σ ones are doubly occupied and the other are empty ($n/l = 1/2$). When Eq. (1) is applied to this state, all factors give 1, except the last one which determines the phase: $\gamma_c = \sum_{j=1}^{N_\sigma} 8\pi j/L = \pi(N_\sigma + 1) \equiv 0$ (π) if N_σ is odd (even). The result is the same for states translated to any other place in the chain, and to linear combinations of these states (in particular with well defined K). This result suggests that choosing appropriately L one might be able to detect the boundary between S and PS states. Another example in which γ_c is easy to calculate is a CDW for even L in which every second state is singly occupied. Eq. (1) gives $\gamma_c = \sum_{i=1}^{L/2} 8\pi i/L = \pi(L+2) \equiv 0$. The same result is obtained if every fourth site is doubly occupied.

The fact that a Berry phase can jump sharply at the boundary between an S phase and a CDW or PS state, can be useful in finite-size diagonalizations, where correlation functions vary smoothly at the transition, and traditional calculations, like those based on bosonization and conformal-field theory results [15], which use the compressibility obtained numerically from the energy necessary to add and remove two particles, may have large finite-size effects. For example, in the phase diagram of the 1D generalized $t-J$ model including a three-site term, a superconducting bubble is predicted inside the PS region, which is an artifact of finite-size effects in the compressibility [16]. Instead, γ_c does not depend on the properties of the system for other densities and has a unique value on the PS state. In other words, although a real phase transition for most systems of interest can only take place in the thermodynamic limit, under certain circumstances, both thermodynamic phases are characterized by different topological quantum numbers, accessible in finite-size systems, and with small size dependence. This was shown in Ref. [10] for a half filled system. Here I calculate γ_c for the following 1D model at quarter filling ($n/l = 1/2$):

$$H(0,0) = -t\sum_{j\sigma} c^\dagger_{j+1\sigma} c_{j\sigma} + U\sum_j n_{j\uparrow} n_{j\downarrow} + V\sum_{j\sigma\sigma'} n_{j+1\sigma} n_{j\sigma'}. \quad (3)$$

I restrict most of the study to $U < 0$. To distinguish the PS state from the one with dominating singlet superconducting correlations at large distances (S), it turns out that the odd N_σ case mentioned above (leading to γ_c(PS)= 0) is the convenient choice. Thus, I took $L = 12$, $N_\uparrow = N_\downarrow = 3$, for the numerical calculations. As expected from the above discussion, I obtain in the region $V > |U| \gg t$, where charge-charge correlations dominate at large distances γ_c(CDW)= 0.

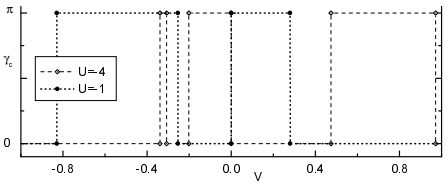

FIG. 2. Charge Berry phase as a function of V for $U = -4$ (dashed line) and $U = -1$ (dotted line).

In the S region, $\gamma_c = 0$ or π depending on the parameters. Fig. 2 illustrates γ_c as a function of V. There are several jumps in γ_c which do not provide information on phase boundaries. In particular, there is always a jump at $V = 0$, and for large $|U|$ there is another one near $V = V_c^1 = -(t^2/|U| - 4t^4/|U|^3)$. To illustrate this jump, consider the case $|U| \gg t, V$. In this limit, it is known that H reduces to a model of N_σ hard-core bosons, representing doubly occupied sites, which can hop to NN positions with matrix elements $t' e^{\pm i(\Phi_\uparrow + \Phi_\downarrow)}$, with $t' = 2t^2/|U|$ ($t' = -2V_c^1$ if fourth order corrections are included), and NN repulsion $V' = 2(2V + t')$. In 1D, each hard-core boson can be mapped into a spinless fermion with an appropriate change in the boundary conditions [17] and this model is equivalent to an XXZ model $\sum_{<ij>\alpha} J_\alpha S_i^\alpha S_j^\alpha$ with N_σ spins up, and $J_x = J_y = 2t'$, $J_z = V'$. Another way to reach the same result is to perform first the unitary transformation $c'_{j\uparrow} = c_{j\uparrow}$, $c'_{j\downarrow} = (-1)^j c^\dagger_{j\downarrow}$ (which in H has the effect of changing the sign of U and $\Phi_\downarrow$ and transforming the NN repulsion in $4V\sum_{<ij>} S_i^z S_j^z$), and then eliminate terms linear in t through a canonical transformation. For $V' = 0$ the model can be solved trivially since the interaction vanishes in the spinless-fermion model. Then, one might draw a picture like Fig. 1 to represent $|g_0(\Phi,\Phi)\rangle$ and again there is a crossing of levels at $\Phi = \pm 2\pi$ of states with the same $K = 0$, but different (singly) occupied (effective) one-particle states $\bar{k}$, leading to an undefined γ_c. However, in contrast to Fig. 1, states with $K = \pm\pi/2$ appear in the low-energy manifold and the ground-state energy as a function of flux Φ displays periodicity in π (with minima at $\Phi = \pi \times$ integer): the anomalous flux quantization characteristic of the S phase [10,12,18]. A small V' leads to a well defined γ_c which depends on the

sign of V'. I also find other jumps of γ_c for $V = 0$ and at other points inside the S phase, which do not provide information on phase boundaries. The physical meaning of the other jumps inside the S phase is not obvious.

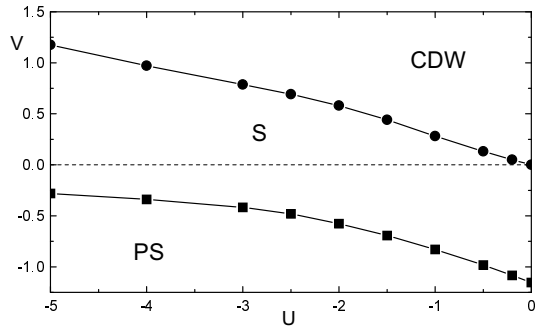

FIG. 3. Phase diagram of the extended Hubbard model Eq. (2) determined through jumps in γ_c, showing the regions of dominating superconducting (S) or charge-density wave (CDW) correlations at large distance, and the region of phase segregation (PS).

Instead, I find that for $|U| \gg t$, the first and the last transition as a function of V agree very well with the limits of the S phase, obtained in the thermodynamic limit through the exact Bethe-ansatz solution of equivalent XXZ models: from the solution of the above mentioned XXZ model [19], we know that PS occurs for $J_z < -J_x$ (ferromagnetic Ising-like case), what in terms of the parameters of Eq. (3) means $V < V_{PS} = -2t^2/|U|$. On the other hand, when $V \gg t^2/|U|$, extending previous ideas [17,18], the entity composed of a doubly occupied site and an empty site right to it can be mapped into a spinless fermion. After an appropriate canonical transformation [18], I find that the resulting spinless fermion model has on-site energy $U - 4t^2/(V - U)$, NN repulsion $V' = 4t^2[1/(V - U) - 1/(3V - U)]$, hopping $t'e^{\pm i(\Phi_\uparrow + \Phi_\downarrow)}$, with $t' = 2t^2/(V - U)$, number of particles $N' = N_\sigma = N/2$ and number of sites $L' = L - N/2$. I have calculated the correlation exponent K_ρ of this model in the thermodynamic limit from its compressibility and charge velocity v_c mapped appropriately to the original model [17]. The energy and v_c were obtained solving the corresponding integral equations of the equivalent XXZ model [19]. I obtain that the boundary $K_\rho = 1$ between dominating S or CDW correlations at large distances corresponds to $J_z \cong 0.25J_x$, leading to $V_{CDW} \cong 0.20|U|$. For $5t < |U| < 10t$, I find that for $L = 12$, the first jump in γ_c with increasing V is above V_{PS} by $\sim 25\%$ and the last jump is above V_{CDW} by $\sim 15\%$. These numbers are reduced by a factor $\sim 2/3$ if fourth order corrections to t' and V' are included. I believe that the remaining discrepancy is due to finite size effects. For smaller values of $|U|$, the first and last jumps in γ_c, shown in Fig. 3 agree with previous numerical calculations of K_ρ and compressibility [18,20]. In correspondence with these calculations, I find an island for high V and small $|U|$ ($V \sim 8t$, $U \sim -2t$, not shown) in which γ_c jumps to π and $K_\rho > 1$. However, using again an appropriate mapping to two XXZ models, I obtain that this is a finite-size effect and that in the thermodynamic limit the system phase separates into two phases with dominating CDW correlations, in agreement with previous suggestions [18]. One phase is the one just described for $V > V_{CDW}$, and in the other one fermion objects composed of one particle and an empty site next to it move with hopping t and NN attraction $V' = -2t^2/V$.

Another limit in which the charge dynamics is equivalent to that of interacting spinless fermions is that of $U = +\infty$. Following similar methods as those already used for $V = 0$ [21], the model can be mapped into the spin polarized case $N_\uparrow = L/2$, $N_\downarrow = 0$, with a shift $\Delta\Phi$ in the BC. Calculating K_ρ in the thermodynamic limit, I find that superconducting correlations dominate for $-2t < V < V_{CDW}$, where up to three digits accuracy $V_{CDW} = -\sqrt{2}t$. For $8 \leq L \leq 16$, γ_c jumps exactly at this value . Instead, γ_c does not detect the transition to the PS state at $V = -2t$ in this case.

In the rest of this Letter, I discuss the spin Berry phase γ_s. As noted earlier [10,11], γ_c is transformed into $\gamma_s + \pi$ (of a different Hamiltonian in general) and vice versa by the transformation $c'_{j\uparrow} = c_{j\uparrow}$, $c'_{j\downarrow} = (-1)^j c^\dagger_{j\downarrow}$. Thus, the definition of γ_s can be generalized to $N_\uparrow \neq N_\downarrow$ in the same way as γ_c, keeping $\Phi_\downarrow = -\Phi_\uparrow$ in the l cycles, and l should be such that $(N_\uparrow - N_\downarrow)l/L$ is an integer. It is also clear from this transformation and what I obtained for γ_c (Eq.(2)) that $\Delta(P_\uparrow - P_\downarrow) = (e/2\pi l)\Delta\gamma_s$ (modulo e/l),where P_σ is the macroscopic polarization of the particles with spin σ. If $N_\uparrow = N_\downarrow$, as in most cases of interest, the previous definition of γ_s with $l = 1$ [10], is valid for any filling, and for 1D systems with spin SU(2) symmetry, γ_s jumps in π when a spin gap Δ_s opens. To show this, I use recent results from continuum limit theory and bosonization [22], which show that the opening of a spin gap is determined by the crossing of the lowest energy states of $\bar{H}$ with total spin $S = 0$ and $S = 1$ within the $\bar{K} = 0$ sector, for periodic (antiperiodic) BC if N_σ is even (odd). These BC correspond to a particular point $\Phi = \Phi_c$ with $\Phi_c = 0$ ($\Phi_c = \pi$) in the trajectory with $\bar{K} = K = 0$ of $H(\Phi, -\Phi)$ used to define γ_s. Usually this point corresponds to the *maximum* of the ground-state energy $E_0(\Phi, -\Phi)$ as a function of Φ, while its minimum lies at $\Phi = \pi$ ($\Phi = 0$) for N_σ even (odd), as expected from the non-interacting limit.

From its definition, it is obvious that to calculate γ_s any (single valued) Φ-dependent gauge can be chosen. I take the representation in terms of the operators $\tilde{c}^\dagger_{j\uparrow} = e^{ij\Phi_c/L}c^\dagger_{j\uparrow}$, $\tilde{c}^\dagger_{j\downarrow} = e^{-ij\Phi_c/L}c^\dagger_{j\downarrow}$, in such a way that in the resulting Hamiltonian $\tilde{H}(\Phi, -\Phi)$, all hoppings are real for

$\Phi = \Phi_c$ (as in $\bar{H}(\Phi_c, -\Phi_c)$). Sufficiently near to Φ_c, one can approximate $\tilde{H}(\Phi, -\Phi) \simeq \tilde{H}(\Phi_c, -\Phi_c) + \frac{\partial \tilde{H}}{\partial \Phi}(\Phi - \Phi_c)$, where the derivative is evaluated at Φ_c. While at $\Phi = \Phi_c$, the SU(2) symmetry is conserved, for $\Phi \neq \Phi_c$ only spin rotations around the quantization axis z are conserved by $\tilde{H}$. Specifically, $\partial \tilde{H}/\partial \Phi$ is equal to the paramagnetic part of the difference between up and down currents, and changes sign under spin rotations in π around the x or y axis. I denote these rotations by R and call $|s\rangle$, E_s ($|t\rangle$, E_t) the lowest singlet (triplet with spin projection zero) eigenstate of $\tilde{H}(\Phi_c, -\Phi_c)$ and its energy. It is known that $R|s\rangle = |s\rangle$, and $R|t\rangle = -|t\rangle$. Since $\partial \tilde{H}/\partial \Phi$ is odd under R, this implies $\langle s|\partial \tilde{H}/\partial \Phi|s\rangle = \langle t|\partial \tilde{H}/\partial \Phi|t\rangle = 0$, However, in general $\langle s|\partial \tilde{H}/\partial \Phi|t\rangle = A \neq 0$, where A can be made real by a suitable change of the phase of one of the states. Near the crossing point between lowest singlet and triplet energies, the ground state $|g_0(\Phi, -\Phi)\rangle$ results from the diagonalization of the low-energy part $\tilde{H}_{LE}$ of $\tilde{H}$ containing the states $|s\rangle$ and $|t\rangle$, which according to the above symmetry arguments, has the form:

$$\tilde{H}_{LE}(\Phi, -\Phi) \simeq \begin{pmatrix} E_S & A(\Phi - \Phi_c) \\ A(\Phi - \Phi_c) & E_T \end{pmatrix}. \qquad (4)$$

It is easy to see that this leads to a jump in π in γ_s when E_T crosses E_S. For example, one can exploit again the gauge invariance to render $\langle s|g_0(\Phi, -\Phi)\rangle$ real and positive for Φ near Φ_c, leading to $|g_0(\Phi_c + 0^+, -\Phi_c - 0^+)\rangle = |g_0(\Phi_c - 0^+, -\Phi_c + 0^+)\rangle$ for $E_S < E_T$ but $|g_0(\Phi_c + 0^+, -\Phi_c - 0^+)\rangle = -|g_0(\Phi_c - 0^+, -\Phi_c + 0^+)\rangle$ for $E_S > E_T$ and no other contribution to γ_s in the vicinity of Φ_c. I obtain numerically in the extended Hubbard model with or without correlated hopping [10] that $\gamma_s = 0$ in the spin gapped phase and therefore $\gamma_s = \pi$ if $\Delta_s = 0$. According to continuum limit theory and renormalization group, in 1D, when $K_\rho > 1$, the opening of a spin gap signals the transition from dominating triplet to singlet superconducting correlations at large distance.

In summary, I have shown the ability of two Berry phases, and topological quantum numbers derived from them, to detect phase transitions, particularly in strongly correlated systems with non-integer number of particles per site, for which no previous applications of Berry phases were considered. I hope that this study will stimulate further research and applications of the Berry phases in phase transitions, particularly in dimensions higher than one, where alternative methods which use results of conformal field theory, or exact solutions [12,14,16–20,22] are not available.

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